Decision step strategy: Applying the general method to a specific problem

Taken from: J. Fleet, F. Goodchild, R. Zajchowski, “Learning for Success”, 2006. See R. Zajchowski for a completed example.

Purpose:

To help learners focus on the process of solving problems, rather than on the mechanics of formula and calculations.

The focus is on correct application of concepts to specific situations. This strategy helps you to increase your awareness of the mental steps you make in problem solving, by “forcing” you to articulate your inner dialogue regarding procedure.

Method:

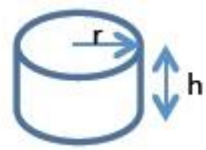
Identify the key decisions that determine what calculations to perform. In lecture, try to record the decision steps the professor uses but may not write down or post.

i. Analyze solved examples, using brief statements focusing on steps you find difficult:
   - What was done in this step?
   - How was it done; what formula or guideline was followed?
   - Why was it done?
   - Any spots or traps to watch out for?

ii. Test run the decision steps on a similar problem, and revise until the steps are complete and accurate.

Example: Decision steps in Calculus for max/min word problems

Problem: A peanuts manufacturer wishes to design a can to hold dry-roasted peanuts. The volume of the cylindrical can is 250 cm^3, and the circular top of the can is made from aluminum while the sides and the bottom are made from stainless steel. If aluminum is twice as expensive as stainless steel, what are the most economical dimensions of the can?

Steps	Solved example
1. Identify Quantity to be maximized/ minimized (Q)	C=Cost per can
2. Diagram when possible (including variables) a. Shapes (perimeter, area, volume) b. Equations to be graphed (axes, levels, distances)	 V = 250 cm^3
3. Make equation for questions using terms from formulas a. Perimeter (P), surface area (S.A.), volume (V) b. Pythagorean relationship c. Sums, differences d. Cost of steel (k), overall cost (C) e. Distance between points - Define variables - Often need to combine equations	$V=\pi r^2 h = 250\ cm^3$ (1) $S.A. = 2(\pi r^2) + 2\pi rh$ (2) = (top+ bottom) + side $C = (2k)\ \pi r^2 + (k)(\ \pi r^2 + 2\pi rh)$ (3) = aluminum top + steel side and bottom
4. Substitute the given volume value into equation (1) to get h(r) and substitute into equation (3) to get C(r).	From (1): $h = \frac{250}{(\pi r^2)}$ From (3): $C = 3k(\pi r^2) + 2k\pi r(\frac{250}{\pi r^2})$ $C = 3k\pi r^2 + \frac{500k}{r}$
5. Set the 1st derivative of overall cost (C) with respect to radius to 0 to find the radius that gives the optimum overall cost	$\frac{dC}{dr} = 6k\pi r - \frac{500k}{r^2} = 0$ Cross out "k" in both terms since it is common in both, rearrange equation, and solve for r: r=2.98 cm and from (1): h=8.947cm
6. Check 2nd derivative to verify the values found for "r" and "h" indeed give a **minimum** cost. (if 2nd derive >0, min. cost is found; if 2nd derive <0, max cost is found)	$\frac{d^2C}{dr^2} = 6k\pi + \frac{500k}{r^3}$ Since the right hand side of the equation can never be negative, r=2.98cm gives the minimum cost.
7. State answer; watch significant figures	The most economical dimensions for the can are r=3.0 cm and h=8.9 cm.

The ‘what’ and the ‘how’

Note that these decision steps try to capture WHAT and especially HOW each step is carried out – including possible alternatives that can be tweaked so that the student is not left wondering how to make the decision needed. Most textbook steps tend to give the WHAT only. For example, these are steps from a calculus textbook:

1. Determine the quantity Q to be maximized or minimized
2. If possible, draw a figure illustrating the problem
3. Write an equation for Q in terms of another variable of the problem
4. Take the derivative of the function in step 3 … etc.

From Washington A.J. (2000). *Basic Technical Mathematics with Calculus* (7^{th} ed.), Addison Wesley Longman.

Decision steps for rational expressions

Math 172. Used with permission.

1. Read question.

2. Make table:
    a. Identify cases (include a third case if total or difference of both cases)
    b. Put equation at top of table
    c. W = r x tor
    d. Total Cost= Cost/person x #of people

3. Fill in columns of table with knowns and unknowns:
    a. Use letters for formulas above for unknowns
    b. If two columns are filled, then do third by algebra
    c. **Watch**! Do previous step carefully!

4. Set up equation:
    a. Sum? Then add rates
    b. Difference? Then subtract rates
        i. **Watch**! Which rate is bigger? Then add to smaller

5. Solve resulting equation for one of the cases

6. Find answer for 'other' case

7. Check by substituting answer into its respective case

8. Write answer in appropriate format

Note:

1. Carefully following these steps should allow you to solve any problem of this kind. If these steps don't quite 'work' adjust them so that they do.
2. As you can see, good decision steps often explain HOW to do a complicated or new step quite carefully. They are much more than just a general approach e.g. "Read question, create table, set up and solve equations"
3. Good decision steps also can – and should – include some 'watch' steps to remind you to be careful in spots where it is easy to make careless errors