Math Problem Solving

Math is about creativity and communicating ideas and their connections. It's *not* just about getting the right answer. Getting stuck and making mistakes are not a sign that you're bad at math; they're a necessary part of the process.

Math takes **time**. To keep going even when it's difficult, you need a **plan** and you need to use your **resources**. You don't have to figure this out on your own!

Our math strategies are supported by scholarly research.

What math profs hope you do

Homework is critical for success in math courses. Problem sets are for **figuring out**, and then **practicing**, new and different ways to solve a type of problem. **The process is what matters.** Thinking through a problem will deepen your understanding and help clarify the questions you will ask peers, TAs, or profs. Below, we offer strategies that will help you get your math homework done effectively.

Learn, then study

- ***Learning*** is improving your understanding of facts, concepts, processes and relationships (i.e., your understanding of course content).
- ***Studying*** is reviewing material you've *previously learned* to increase your recall of course content.

We recommend that you focus on *learning* during your homework time, and save ***studying*** for when you need to prepare for exams and tests. Don't try to do both at once.

Spend enough time on your math

- See how you do by putting in **8-10 hours, per week,** on **each course** (this time **includes** time you spend in class, labs, doing homework, etc.).
- Spread out your work; do some math every day. It will add up.
- Keep up with the homework. Concepts later in the term build on the ones from earlier in the term, so get help right away if you start to struggle.
- If you have trouble **managing your time** or **staying focused**, make an appointment with an **academic skills specialist**; we can help!

Do your homework

1. Review your notes from the day's lectures:

- Read over your notes from each class of the day; this habit supports your memory of the content, which is helpful when it's time to study.

- Fill in gaps in your notes and identify what you don't understand.
- In three or four points, summarize each lecture (don't just copy it) to use as study notes.
- This process might take 10-15 minutes for each lecture's notes.

2. Complete your assignments:

- Keep up to date with assignments; aim to finish one day ahead of the due date.

3. Preview the next day's lectures:

- Before class, preview the lecture notes / assigned problem set to anticipate the lecture's main ideas.
- Skim any assigned text to get an overview of the content: read the chapter introduction, summary, glossary and review questions, then return to the chapter's beginning and read for more detail. Note new or complex content, and listen for it in the lecture.

4. Do a weekly review:

- Schedule a regular block of time to review your summarized notes from the week's lectures, readings, concepts and key problem types. This might take 20-30 minutes per course.
- Pay attention to what you don't know; set a goal of figuring it out over the next week.

Learn effectively

Before you start your homework questions, **review** your class notes and/or the relevant textbook chapter, and **identify** the key concepts that they describe. Try working **a sample problem** from your notes or text, without looking at the solution, to see if you understand the idea. Then try the homework problems.

Be thorough. Don't just rush through problem sets.

- Take a systematic approach (e.g., **Polya's problem solving techniques**).
- Read and define the problem first; it takes time, but it's worth it.
- Think of **problems as a way to communicate,** from the problem-setter to you. Ask: what do I know (givens)? What can I do? Are there clues or keywords in the problem that point to a particular concept?
- Look for and understand the underlying concept (the "why" or "big picture") of each question, not just the procedure for solving it.
- Diversify your thinking; there's often **more than one way to solve** a problem.
- **Model the problem:** draw it, talk it out, use analogies, change something (e.g., the scale), or ask "what if…" as ways to see the problem in a new way.
- Produce a complete and well-reasoned solution, not a superficial one.
- Aim for accuracy before you aim for speed.
- Spend time on challenging questions, not just familiar ones.

Recognize repeat concepts.

- Most math courses ask you to do hundreds of problems, but these problems usually fall under just a few concepts that you'll revisit in different forms over the term.
- Learn to identify and understand these few concepts and their relationships to each other, and recognize them when they take different forms.
- The learning objectives in your course syllabus often tell you what the key concepts are.

Self-assess and reflect.

- Monitor your thought process while solving problems (try our decision steps).
- **Work out loud;** notice what strategies you're using and why.
- Reflect on how you present your thinking in your solution. Is it clear and purposeful?
- Monitor your progress; change your approach if you need to.
- Ask "does this make sense?" and "did I solve the problem/answer the question?"
- Check the reasonableness of your answer.

Don't give up.

- Expect math to be a challenge and to take time. Keep trying.
- Mistakes are valuable! They aren't a sign that you're bad at math; they're a necessary part of the process.
- Identify **where you get stuck** and, if you can, why. **Prepare questions** to bring to your TA / prof / help desk.
- Take a break when you feel frustrated.
- Be optimistic. The problem *does* have a solution.
- Don't assume you're not a "math person."

Manage your progress

Follow the steps and links below to set your course learning goals and progress toward them, while also building your self-regulation and thinking strategies.

- First, **make a plan** for getting unstuck before you even start working on problems.
- **Identify and organize** the course's key concepts and problem types.
- **Understand** these key concepts (using the concept summary tool) and problem types (using the **decision steps tool**).
- **When you get stuck**, remember that it's just part of learning. Use one or more of your planned strategies or resources to try getting unstuck.
- **Track** your progress every week.

Get unstuck

In any class, including math class, you *should* get stuck. If you're not getting stuck, you're not learning.

It's helpful to **plan in advance** what to do when you're stuck, and to keep this plan near you when you do your homework or study. That way, when you're stuck, you can look at your plan and choose

something to do about getting unstuck right away, instead of panicking or feeling discouraged and giving up.

Know your resources

Make a list of where you will go to find information about the math concepts you're working on. Take some time to think about and write down resources and supports that will help you learn. Ask upper-year students or your TA for ideas for this list.

- **Where to find information:** Figure out the key places to find information about what you're working on:
    - course notes
    - the course textbook
    - different textbooks
    - problem solution books (books with many worked examples of problems)
    - websites that display solved problems in a variety of ways
    - websites that refresh your memory of an underlying concept that you need to grasp before you can work on the problem you're stuck on.
- **Who can help:**
    - your prof / TA; figure out when their office hours are
    - people in your class that you might want to study with
    - the **Math Help Desk**; note when it offers help for your course
    - ask your prof early on for suggestions for additional resources that may help you learn content or study for the exam.

Ask effective questions

Prepare to ask questions: TAs, professors, and peers generally want you to succeed and will welcome questions.

- If you don't know where to start with a problem, you can still explain in general what you know about the concept, and what you're thinking of doing.
- If you're stuck in the middle of a problem, but know what to do after that point, make up an answer for the step you're stuck on and use it to solve the rest of the problem. Then get help. Your attempt at a solution will get you better feedback from your TA/professor and will mean more than no attempt at all.

If you feel really lost, it might help to review (or learn) content that the current problem is based on. Talk to your TA or prof about what foundational concepts come *before* this particular concept / problem; look them up in a textbook or on a reliable website such as Khan Academy to make sure you understand them, before returning to the current concept / problem.

Other helpful ideas

Make math more social to boost your skills, motivation, and confidence. Work through problems as a group, share resources, talk through solutions, and explain concepts to each other.

Become more **comfortable with uncertainty** and the unknown. Try thinking of solving math problems as a game or an interesting puzzle. Play around with creative approaches.

Stop if you become overly frustrated; do something else and try again later with a clearer mind.

Check out other SASS resources: **academic skills resources**, **subject-specific academic resources**, **workshops**, and **appointments**.

Next, identify and organize the course's key concepts and problem types.

Identify and organize concepts and problem types

Instructions

Identify and organize key concepts and problem-types in a course by using the course learning tracker or a similar format of your own. Getting this tool "finished" is not the goal. What's important is that you think about the course's key concepts and how they relate to the types of problems you're being asked to solve.

Identify the key concepts, and the problem types for each key concept, by:

- reading the learning objectives in the course outline or course description
- checking the syllabus and weekly reading list
- referring to the textbook's table of contents and headings within a chapter (focus on how concepts and practice problems are organized and related to each other)
- thinking about the key formulas (which are often related to key concepts), and common aspects of the problems you are solving
- referring to the lecture outline to identify recurring themes
- referring to learning objectives and topics from lectures and other materials
- past years' final exams, if available.

Why focus on concepts?

Many students try to jump straight into solving problems without working on understanding the concepts that are being applied in those problems. They try to look for specific formulas that match specific problems and end up memorizing too much information with almost no hierarchy or connection.

This matching of formulas with problem types can work in the short term. It is possible to solve a number of problems very quickly by memorizing a few formulas and solutions, and you may be able to keep this up for a unit or two, but it won't work over the course of an entire semester, and that's what we care about in university.

In university math courses:

- You have **far too many** formulas and solutions to try to memorize.

- Professors will often choose exam questions that cannot be solved by referring simply to a specific memorized formula and solution. They want to test whether you really can **recognize, understand and apply** the underlying **concept**.
- Courses, and the units within those courses, are sequential. Like a set of building blocks, today's concept is **built on previously learned concepts**. If one of those previous concepts is missing in your understanding, the whole structure will fall down.

The mental work involved in understanding key concepts helps clarify ideas and shift the conceptual information from working memory to long-term memory, which helps you recall material for exams and future courses.

Next, build your understanding of key concepts and problem types.

Understand concepts

Build your understanding of key concepts (e.g., the slope of a line) using the **concept summary** tool. Build your understanding of problem types / applications (e.g., calculate the slope from a graph) that fall under each concept using the **decision steps** tool.

When you use the **concept summary** and **decision steps** tools, make a note in the "notes" section of **the course learning tracker** of where you get stuck or what you don't understand. You can use these notes as a springboard for conversations with your profs, TAs, tutors, and peers that are targeted specifically to your learning needs.

Keep in mind that while the final products of these learning tools—your completed concept summaries and decision steps—are important and will make valuable study aids, the *process* of thinking, struggling, and questioning while working with these tools is essential for achieving a deep understanding.

Concept summary

Download the concept summary tool (PDF).

(Fleet, Goodchild, & Zajchowski, 2006)

Concepts are general organizing ideas. Often, a course will cover just a few key concepts, along with their many applications. Learn and understand the small amount of information essential to each concept. If in doubt, ask the professor what is important for you to "get."

Instructions

Choose a key concept from your course (identified when you filled in **the course learning tracker**), and use the following five categories as a guide for building your understanding of that concept:

1. title
2. key equations
3. definitions of each term in the equations

4. additional information (e.g., sign conventions, reference values, the meaning of zero values, cases where the concept doesn't apply, relevant knowns/unknowns)
5. your own example or explanation.

Note: An example of a concept summary for PHYS 117 is **included in the PDF**.

Decision steps

Download the decision steps tool (PDF).

(Fleet, Goodchild, & Zajchowski, 2006)

This tool is helpful for any applied problem-solving situation (e.g. mathematics, physics, statistics, accounting). To help learners focus on the process of solving problems, rather than on the mechanics of formula and calculations, the focus is on correctly applying concepts to specific situations. This strategy helps you to increase your awareness of the mental steps you make in problem solving, by "forcing" you to articulate your inner dialogue as you go.

Instructions

To build your understanding of the various applications (or problem-types) of key concepts:

- Choose a problem type in your course (identified when you filled in **the course learning tracker**), and analyze / build decision steps for a solved example (from a lecture, from your homework, from a study guide) by answering **what** was done, **how** it was done, and **why** was it done for each step.
- During the lecture or when you read course notes, **focus on the process of solving the problem, instead of on the computation**. When your professor is lecturing, listen to their comments on how steps are **linked** from one to another. This helps you identify the decision steps that lead to correct application of a concept. Ask yourself, "why did I move from this step to this step?"
- Test run these decision steps by using them to solve a similar problem. Usually, your initial decision steps will be incomplete, and require revision.

Note: An example of the decision steps tool applied to PHYS 117 is **included in the PDF**.

Track your progress

Tracking your progress is vital to **understanding the main concepts** of your math courses. Why? It will help you:

- turn your attention to the higher levels of thinking required at this level
- build the essential skill of self-regulation
- monitor what you need to know and what you currently know, and find ways to close the gap
- stay on top of your course work
- get unstuck, since it helps you ask more specific questions of your professor or TA.

Two tools can help you track your progress: the **course learning tracker** and the **issue bin**. You can fill these in with the **course concepts** (**link to** "Identify and organize concepts" tab) you identified already.

The course learning tracker

Download the course learning tracker (PDF).

The course learning tracker is a simple **chart** that you can update weekly as you attend lectures and complete homework, problem sets, and assignments. It's useful to set up a table like this somewhere you can access it easily—in the front or back of a notebook for a course, or somewhere prominent on your note-taking app. That way whenever you need to note down something about your progress you'll be able to do so easily.

This chart should have four columns:

1. **Key concepts** in the course: you can add to (or delete from) the content of this column as you progress through the course and gain a clearer idea of the main ideas. Add a brief description of each concept in this column.
2. **Concept checklist**: in this column, add a check mark against each of the concepts you listed in the "key concept" column if you understand them well enough to explain them to someone else.
3. **Problem checklist**: add a check mark here if you can apply the concept or do the problem type on your own and fully explain the process.
4. **Notes while learning**: here, add anything that helps you stay organized and track your progress in understanding these concepts and problem types. You might note questions that you want to ask your prof, or flag concepts that are especially important or are likely to come up on an exam.

Issue bin

Download the issue bin (PDF).

The issue bin is similar to the course tracker, but you can use it to describe more general progress. This tool may be particularly useful if you are struggling to define or list the concepts you need to learn in a particular course.

To use the issue bin, create a notetaking file for each course. Since the issue bin can get a little messy, and you'll constantly write and rewrite it, it's easier to use software rather than paper to complete it. Add a 2×2 table to your notetaking file, then add a subtitle and information for each of the four squares in the table as shown below.

1. Questions or misconceptions that I need to address
2. Held: high priority questions / misconceptions I've tried to resolve but can't yet.
3. Parked: low priority questions / misconceptions I've tried to resolve but can't yet.
4. Resolved questions / misconceptions

Write each concept or theory that you don't yet understand in square A; move it to the other bins as you resolve it. You might update the issue bin every time you work on a particular course as you

discover or resolve issues, or you might return to the bin weekly to update it with your progress; it's up to you.

(In the spirit of academic integrity, we'd like to thank Dr. Alan Ableson from the Faculty of Engineering & Applied Science for sharing this resource.)

Over the course of the semester, as you attend lectures and complete problem sets and assignments, continue to track your progress:

- monitor key concepts and add, merge, or separate concepts as necessary
- make, or add to, existing **concept summaries** and **decision steps**.

Study for tests and exams

In math-based courses, your goal should be to **focus on solving problems**, not on reading. For example, if you have six hours to study for an upcoming test, spend one hour reading and five hours doing problems.

Use problem sets effectively

- **Do problems to mastery.** Once you've mastered one kind of problem, don't worry if you haven't finished every single problem in the set–move on to the next type, or apply what you've learned in a different context.
- **Use the answer key strategically.** Avoid looking at the answer key while you work on a problem, but don't do problems without checking to see if your answer is correct.
- **Ask for help when you need it.** Use the decision steps and concept summary tools to help communicate what you know and where you got stuck so that you can ask specific questions.
- **Work backwards.** For problems where you are given the answer but don't know the starting point, begin at the end and work backwards to undo the problem step by step.
- **Use images.** What can you draw to help yourself understand and solve the problem? Can you make a mental picture or otherwise visualize this problem?

Study techniques

- **Interleaving:** Mixing up problem types supports your learning. The aim is to arrange problems so that consecutive problems cannot be solved by using the same strategy. **Retrieval practice** has a guide that can help you get started; see **interleaved mathematics practice**.
- **Self-testing** helps you anticipate different kinds of difficult problems for exam preparation, and solve some practice problems to test yourself. Don't wait until the night before the exam! The more frequently you self-test, the better your learning and memory will be.
- **Explaining to and/or teaching others** are great ways to make sure you're thinking aloud, describing the problem, and working with others. Use **study groups** to compare completed solutions to assigned problems. Teaching someone is very effective learning and study technique.

For more study strategies, see our **test and exam preparation** section.

Resources at Queen's

Your professor/TA: Using the course syllabus, figure out when your professor's office hours are, and whether they will include online meetings, live seminars or Q&A sessions as part of their course. If there is nothing formal set up, reach out and ask for the best way to contact the instructor. Contact the professor and/or TA as early as possible in the course to introduce yourself and open up the lines of communication. This will make it easier to later ask for help with assignments or with finding additional resources.

Your peers: Everybody will struggle with *something* in their courses at Queen's. Use social media groups, course unions, and discussion boards within courses to build a network of people that you might want to study with. The earlier in the term you do this, the more you will benefit from these connections!

Math Help Centre: The **Math Help Centre** is a valuable source of support.

Tutors: The **Math Department** has a list of tutors—mostly graduate students and professional teachers—who may be able to help you. You will have to pay for their services, so you many want to make this your last resort, rather than your first.

Academic skills specialists at SASS can work with you one-on-one to discuss your current approaches to math courses, and choose strategies that are likely to help.

External resources

1. **Khan Academy** offers free practice problems, explainers, and other help through their website and on YouTube.
2. MIT's **Single-Variable Calculus Course** contains clear explanations of many fundamental calculus concepts that you might encounter in the first year or two at Queen's.
3. ***The Essence of Mathematics Through Elementary Problems*** is a free PDF book that will teach you many university basics.
4. ***Advanced Problems in Mathematics: Preparing for University*** is another free PDF Book that has great resources for Grade 12 students making the leap to university courses.
5. **McMaster University's academic resources website** features three videos on problem solving:
    - **Problem Solver I**, general ideas
    - **Problem Solver II**, differences in applying concepts vs. formula chasing
    - **Problem Solver III**, applying the decision steps strategy

We encourage you to further explore this topic via the articles and books below:

Boaler, J. (2016). *Mathematical Mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching*. San Francisco, CA: Jossey-Bass, Wiley.

Fleet, J., Goodchild, F., & Zajchowski, R. (2006). ***Learning for success**: Effective strategies for students (4th Edition).* Thomson Nelson.

The Learning Scientists. (2020, November 14). *How Students Can Use Interleaving, Elaboration, Dual Coding, and Concrete Examples*. Retrieved from **https://www.learningscientists.org/learning-scientists-**

podcast/2018/3/21/episode-15-how-students-can-use-interleaving-elaboration-dual-coding-and-concrete-examples?rq=problem%20solving.

The Learning Scientists. (2020, November 14). https://www.learningscientists.org/learning-scientists-podcast/2017/10/4/episode-4-spaced-practice?rq=problem%20solving.